Lomonosov Moscow State University

Master's Admission Exam Program

«Geometry and Quantum Fields»

1. Admission exam program

The program of the admission exam is compiled in accordance with the requirements of the federal state educational standard of higher education, presented to the preparation of applicants to the master's program in the field of "Mathematics". The program contains a list of topics, the mastery of which is necessary for successful passing of the entrance exam and a list of recommended literature.

2. List of topics required to pass the admission exam

1. Functions of one and several variables. Continuity of the function. Properties of a continuous function.
2. Functions of many variables, the geometric meaning of a total differential function. Sufficient differentiability conditions. Gradient.
3. Conditions for equality of mixed partial derivatives.
4. The definite integral. Integrability of a continuous function. A primitive continuous function.
5. Implicit functions. Existence, continuity and differentiability of implicit functions.
6. Numerical series. Convergence of series. The Cauchy convergence criterion. Sufficient signs of convergence.
7. Absolute and conditional convergence of the series. Properties of absolutely convergent series. Multiplication of series.
8. Power series in the real and complex domain. Radius of convergence, properties of power series (directed integration, differentiation). Decomposition of elementary functions.
9. Fourier series. Sufficient conditions for the representability of a function by a Fourier series.
10. Ostrogradsky and Stokes theorems. Divergence. A whirlwind.
11. Linear spaces, their subspaces. Basis. Dimension. The rank theorem. A system of linear equations. Geometric interpretation of a system of linear equations. A fundamental system of solutions to a system of homogeneous linear equations. The Kronecker-Capelli theorem.
12. Bilinear and quadratic functions and forms in linear spaces and their matrices. Bringing to a normal appearance. Law of inertia.
13. Linear transformations of linear space, their assignment by matrices. Characteristic polynomial in a linear transformation. Eigenvectors and eigenvalues, the connection of the latter with characteristic roots.
14. Euclidean space. Orthonormal bases. Orthogonal matrices. Symmetric transformations. Reduction of the quadratic shape to principal axes.
15. Groups, subgroups, adjacency classes, Lagrange theorem. The order of the element. Cyclic groups.

16. Group actions on sets, orbits, and stabilizers.
17. Normal subgroups, factor groups. The homomorphism theorem.
18. Differential equation of the first order. Existence and uniqueness theorem.
19. Second-order linear differential equation. Linear homogeneous equation. Linear dependence of functions. A fundamental system of solutions. Vronsky's determinant. Nonhomogeneous linear equation.
20. Linear differential equation with constant coefficients: homogeneous and nonhomogeneous.
21. Functions of a complex variable. Cauchy-Riemann conditions.
22. Elementary functions of a complex variable and the conformal maps they give.
23. Cauchy's theorem on the closed-loop integral. Cauchy integral. Taylor series.
24. Laurent series. A pole and an essentially singular point. Deductions.
25. The concept of a random variable and a random process. Probability. Mathematical expectation, variance.
26. Dynamics of a material point. Newton's laws.
27. Dynamics of a material point. Conservation laws.
28. Movement in a centrally symmetric field. Kepler's laws.
29. Lagrange function and Euler-Lagrange equations of a system of material points. Integrals of motion.
30. Rigid body dynamics. The inertia tensor.
31. Oscillations of systems with one and many degrees of freedom. Free and forced vibrations.
32. Hamilton’s canonical equations. Phase space and Poisson brackets.
33. Fundamentals of the special theory of relativity. Lorentz transformations.
34. Electrostatic field. Coulomb's law. The Gauss theorem.
35. Static magnetic field. The Biot-Savart-Laplace law.
36. Maxwell's equation in a vacuum. Scalar and vector potentials.
37. The energy of the electromagnetic field. The Poynting vector.
38. Electromagnetic waves.
39. Motion of a charged particle in an electromagnetic field.
40. Postulates of quantum mechanics. Wave function.
41. The uncertainty principle.
42. Description of the evolution of quantum mechanical systems. Schrödinger equation. Stationary states.
43. Linear quantum harmonic oscillator.
44. Angular momentum. Addition of Angular Momenta.
45. Movement in the central field. The hydrogen atom: wave functions and energy levels.
46. Systems of identical particles. Bosons and fermions. Pauli's principle.
47. Theory of elastic scattering. Born approximation.

3. Literature

- J. J. Sakurai. Modern Quantum Mechanics. Rev. ed. Addison-Wesley Pub., ISBN 0-201-53929-2, 1994, 513 p.
- R. Shankar. Principles of Quantum Mechanics. Springer, ISBN: 9781461576754, 2013, 612 p.
- S. Leng, Complex Analysis. Springer, ISBN 0-387-98592-1, 1999, 505 p.

- D. Morin, Introduction to Classical Mechanics: With Problems and Solutions. Cambridge University Press, ISBN: 978-0521876223, 2008, 734 p.
- D. J. Griffiths, Introduction to Electrodynamics. Cambridge University Press, ISBN: 978-1108420419, 2017, 599 p.
- D. J. Griffiths, Introduction to Quantum Mechanics. Cambridge University Press, ISBN: 978-1-107-18963-8, 2018, 646 p.